Reference: Simply-typed λ-calculus

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Figure 1: Statics of the simply-typed λ-calculus (with numbers)

Var
$$\frac{}{\Gamma, x : \sigma \vdash x : \sigma}$$

Num
$$\frac{n \in \mathbb{N}}{\Gamma \vdash \mathsf{num}[n] : \mathsf{Num}}$$

Plus
$$\frac{\Gamma \vdash e_1 : \mathsf{Num} \qquad \Gamma \vdash e_2 : \mathsf{Num}}{\Gamma \vdash \mathsf{plus}(e_1; e_2) : \mathsf{Num}}$$

Times
$$\frac{\Gamma \vdash e_1 : \mathsf{Num} \qquad \Gamma \vdash e_2 : \mathsf{Num}}{\Gamma \vdash \mathsf{times}(e_1; e_2) : \mathsf{Num}}$$

Let
$$\frac{\Gamma \vdash e_1 : \sigma_1 \qquad \Gamma, x : \sigma_1 \vdash e_2 : \sigma_2}{\Gamma \vdash \mathsf{let}(e_1; x.\, e_2) : \sigma_2}$$

Unit
$$\frac{}{\Gamma \vdash \langle\rangle : \mathbf{1}}$$

Prod
$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \langle e_1, e_2 \rangle : \tau_1 \times \tau_2}$$

Proj-1
$$\frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_1(e) : \tau_1}$$

Proj-2
$$\frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_2(e) : \tau_2}$$

Abort
$$\frac{\Gamma \vdash e : \mathbf{0}}{\Gamma \vdash \mathsf{abort}(e) : \tau}$$

Inl
$$\frac{\Gamma \vdash e : \tau_1}{\Gamma \vdash \mathsf{inl}(e) : \tau_1 + \tau_2}$$

Inr
$$\frac{\Gamma \vdash e : \tau_2}{\Gamma \vdash \mathsf{inr}(e) : \tau_1 + \tau_2}$$

Case
$$\frac{\Gamma \vdash e : \tau_1 + \tau_2 \qquad \Gamma, x : \tau_1 \vdash e_1 : \tau \qquad \Gamma, y : \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \mathsf{case}(e; x.\, e_1; y.\, e_2) : \tau}$$

Lam
$$\frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x : \sigma.\, e : \sigma \to \tau}$$

App
$$\frac{\Gamma \vdash e_1 : \sigma \to \tau \qquad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1(e_2) : \tau}$$

Figure 2: Dynamics of the simply-typed λ-calculus

$$\textsc{Val-Unit}\quad \frac{}{\langle\rangle\ \mathsf{val}}$$

$$\textsc{Val-Pair}\quad \frac{}{\langle e_1, e_2\rangle\ \mathsf{val}}$$

$$\textsc{Val-Inl}\quad \frac{}{\mathsf{inl}(e)\ \mathsf{val}}$$

$$\textsc{Val-Inr}\quad \frac{}{\mathsf{inr}(e)\ \mathsf{val}}$$

$$\textsc{Val-Lam}\quad \frac{}{\lambda x : \tau.\, e\ \mathsf{val}}$$

$$\textsc{Val-Num}\quad \frac{n \in \mathbb{N}}{\mathsf{num}[n]\ \mathsf{val}}$$

$$\textsc{D-Plus}\quad \frac{n_1 + n_2 = n}{\mathsf{plus}(\mathsf{num}[n_1]; \mathsf{num}[n_2]) \longmapsto \mathsf{num}[n]}$$

$$\textsc{D-Plus-1}\quad \frac{e_1 \longmapsto e_1'}{\mathsf{plus}(e_1; e_2) \longmapsto \mathsf{plus}(e_1'; e_2)}$$

$$\textsc{D-Plus-2}\quad \frac{e_1\ \mathsf{val} \qquad e_2 \longmapsto e_2'}{\mathsf{plus}(e_1; e_2) \longmapsto \mathsf{plus}(e_1; e_2')}$$

$$\textsc{D-Let}\quad \frac{}{\mathsf{let}(e_1; x.\, e_2) \longmapsto e_2[e_1/x]}$$

$$\textsc{D-Proj-Tuple-1}\quad \frac{}{\pi_1(\langle e_1, e_2\rangle) \longmapsto e_1}$$

$$\textsc{D-Proj-Tuple-2}\quad \frac{}{\pi_1(\langle e_1, e_2\rangle) \longmapsto e_2}$$

$$\textsc{D-Proj-1}\quad \frac{e \longmapsto e'}{\pi_1(e) \longmapsto \pi_1(e')}$$

$$\textsc{D-Proj-2}\quad \frac{e \longmapsto e'}{\pi_2(e) \longmapsto \pi_2(e')}$$

$$\textsc{D-Abort-1}\quad \frac{e \longmapsto e'}{\mathsf{abort}(e) \longmapsto \mathsf{abort}(e')}$$

$$\textsc{D-Case-Inl}\quad \frac{}{\mathsf{case}(\mathsf{inl}(e); x.\, e_1; y.\, e_2) \longmapsto e_1[e/x]}$$

$$\textsc{D-Case-Inr}\quad \frac{}{\mathsf{case}(\mathsf{inr}(e); x.\, e_1; y.\, e_2) \longmapsto e_2[e/y]}$$

$$\textsc{D-Case-1}\quad \frac{e \longmapsto e'}{\mathsf{case}(e; x.\, e_1; y.\, e_2) \longmapsto \mathsf{case}(e'; x.\, e_1; y.\, e_2)}$$

$$\textsc{D-App-1}\quad \frac{e_1 \longmapsto e_1'}{e_1(e_2) \longmapsto e_1'(e_2)}$$

$$\textsc{D-Beta}\quad \frac{}{(\lambda x : \tau.\, e_1)(e_2) \longmapsto e_1[e_2/x]}$$