Problem Sheet 3

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The following questions are about the dynamics of numbers and strings.

1. Draw derivations that justify the following transitions.
   (i) $\mathsf{plus}(\mathsf{num}[1]; \mathsf{num}[1]) \longmapsto \mathsf{num}[2]$
   (ii) $\mathsf{times}(\mathsf{plus}(\mathsf{num}[1]; \mathsf{num}[1]); \mathsf{num}[2]) \longmapsto \mathsf{times}(\mathsf{num}[2]; \mathsf{num}[2])$
   (iii) $\mathsf{len}(\mathsf{let}(\mathsf{str}[\text{‘a’}]; v.\, \mathsf{cat}(v; \mathsf{str}[\text{‘b’}]))) \longmapsto \mathsf{len}(\mathsf{cat}(\mathsf{str}[\text{‘a’}]; \mathsf{str}[\text{‘b’}]))$
2. Write down transition sequences that justify the following multi-step transitions.
   (i) $\mathsf{times}(\mathsf{plus}(\mathsf{num}[1]; \mathsf{num}[1]); \mathsf{num}[2]) \longmapsto^* \mathsf{num}[4]$
   (ii) $\mathsf{times}(\mathsf{len}(\mathsf{let}(\mathsf{str}[\text{‘a’}]; v.\, \mathsf{cat}(v; \mathsf{str}[\text{‘b’}]))); \mathsf{num}[2]) \longmapsto^* \mathsf{num}[4]$
3. Are the following terms well-typed? Write down transition sequences that reduce them to values.
   (i) $\mathsf{let}(\mathsf{str}[\text{‘a’}]; z.\, \mathsf{plus}(\mathsf{len}(z); \mathsf{len}(z)))$
   (ii) $\mathsf{let}(\mathsf{len}(\mathsf{str}[\text{‘a’}]); z.\, \mathsf{plus}(z; z))$
   (iii) $\mathsf{plus}(\mathsf{let}(\mathsf{len}(\mathsf{str}[\text{‘a’}]); z.\, \mathsf{plus}(z; z)); \mathsf{num}[1])$
4. The rules D-Plus-1 and D-Plus-2 of the dynamics enforce that e_1 is evaluated *before* e_2 when computing the value of $\mathsf{plus}(e_1; e_2)$. Propose alternative versions of these rules that evaluate e_2 before e_1. Would you expect your rules to affect the final value that is returned?
5. Prove that if e val then either $\vdash e : \mathsf{Num}$ or $\vdash e : \mathsf{Str}$.
6. Prove that multi-step transitions are transitive, i.e. that the following rule is admissible:

$$\frac{e_1 \longmapsto^* e_2 \qquad e_2 \longmapsto^* e_3}{e_1 \longmapsto^* e_3}$$

   [Hint: perform an induction on the premise $e_1 \longmapsto^* e_2$.]
7. ($*$) Complete the proof of preservation.
8. Complete the proof of progress.
9. (Hard, trick, highly optional.) We proved preservation by induction on $e \longmapsto e'$, while we proved progress by induction on $\vdash e : \sigma$. Why did we make that choice? Could we have performed an induction on $\vdash e : \sigma$ for both? Discuss.