

## REFERENCE: PCF

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Figure 1: Statics of PCF

$$\begin{array}{c}
\text{VAR} \\
\hline
\Gamma, x : \sigma \vdash x : \sigma \\
\\
\text{LAM} \\
\frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x : \sigma. e : \sigma \rightarrow \tau} \\
\\
\text{IFZERO} \\
\frac{\Gamma \vdash e : \text{Nat} \quad \Gamma \vdash e_0 : \tau \quad \Gamma, x : \text{Nat} \vdash e_1 : \tau}{\Gamma \vdash \text{ifz}(e; e_0; x. e_1) : \tau} \\
\\
\text{ZERO} \\
\hline
\Gamma \vdash \text{zero} : \text{Nat} \\
\\
\text{APP} \\
\frac{\Gamma \vdash e_1 : \sigma \rightarrow \tau \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1(e_2) : \tau} \\
\\
\text{Succ} \\
\frac{\Gamma \vdash e : \text{Nat}}{\Gamma \vdash \text{succ}(e) : \text{Nat}} \\
\\
\text{Fix} \\
\frac{\Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \text{fix}(x : \tau. e) : \tau}
\end{array}$$

Figure 2: Dynamics of PCF

$$\begin{array}{c}
\text{VAL-ZERO} \\
\hline
\text{zero val} \\
\\
\text{VAL-Succ} \\
\frac{e \text{ val}}{\text{succ}(e) \text{ val}} \\
\\
\text{VAL-LAM} \\
\hline
\lambda x : \tau. e \text{ val} \\
\\
\text{D-Succ} \\
\frac{e \mapsto e'}{\text{succ}(e) \mapsto \text{succ}(e')} \\
\\
\text{D-APP-1} \\
\frac{e_1 \mapsto e'_1}{e_1(e_2) \mapsto e'_1(e_2)} \\
\\
\text{D-BETA} \\
\hline
(\lambda x : \tau. e_1)(e_2) \mapsto e_1[e_2/x] \\
\\
\text{D-FIX} \\
\hline
\text{fix}(x : \tau. e) \mapsto e[\text{fix}(x : \tau. e)/x] \\
\\
\text{D-IFZ-1} \\
\frac{e \mapsto e'}{\text{ifz}(e; e_0; x. e_1) \mapsto \text{ifz}(e'; e_0; x. e_1)} \\
\\
\text{D-IFZ-ZERO} \\
\hline
\text{ifz}(\text{zero}; e_0; x. e_1) \mapsto e_0 \\
\\
\text{D-IFZ-Succ} \\
\frac{\text{succ}(e) \text{ val}}{\text{ifz}(\text{succ}(e); e_0; x. e_1) \mapsto e_1[e/x]}
\end{array}$$