Problem Sheet 6

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The following questions are about call-by-name (CBN) and call-by-value (CBV).

1. Show that the STLC+print term $e \stackrel{\text{def}}{=} e_1(e_2)$ is well-typed. Then, reduce it to a value twice: once using the CBN, and once using the CBV dynamics. Feel free to skip ε in transitions that do not print anything.

$$\begin{aligned} u &\stackrel{\text{def}}{=} \lambda x : \mathsf{Num}.\, \mathsf{print}(\text{`batman'}; \mathsf{plus}(x; \mathsf{num}[1])) \\ e_1 &\stackrel{\text{def}}{=} \lambda f : \mathsf{Num} \to \mathsf{Num}.\, f(f(\mathsf{num}[2])) \\ e_2 &\stackrel{\text{def}}{=} \mathsf{print}(\text{`na'}; u) \\ e &\stackrel{\text{def}}{=} e_1(e_2) \end{aligned}$$

Solution:

$$\begin{aligned} e_1(e_2) &\xmapsto{\varepsilon}_{\mathrm{n}} e_2(e_2(\mathsf{num}[2])) && \text{by D-Beta} \\ &\xmapsto{\text{na}}_{\mathrm{n}} u(e_2(\mathsf{num}[2])) && \text{by D-Print and D-App-1} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} \mathsf{print}(\text{`batman'}; \mathsf{plus}(e_2(\mathsf{num}[2]); \mathsf{num}[1])) && \text{by D-Beta} \\ &\xmapsto{\text{batman}}_{\mathrm{n}} \mathsf{plus}(e_2(\mathsf{num}[2]); \mathsf{num}[1]) && \text{by D-Print} \\ &\xmapsto{\text{na}}_{\mathrm{n}} \mathsf{plus}(u(\mathsf{num}[2]); \mathsf{num}[1]) && \text{by D-Plus-1 and D-Plus-1} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} \mathsf{plus}(\mathsf{print}(\text{`batman'}; \mathsf{plus}(\mathsf{num}[2]; \mathsf{num}[1])); \mathsf{num}[1]) && \text{by D-Plus-1 and D-Beta} \\ &\xmapsto{\text{batman}}_{\mathrm{n}} \mathsf{plus}(\mathsf{plus}(\mathsf{num}[2]; \mathsf{num}[1]); \mathsf{num}[1]) && \text{by D-Plus-1 and D-Print} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} \mathsf{plus}(\mathsf{num}[3]; \mathsf{num}[1]) && \text{by D-Plus-1 and D-Plus} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} \mathsf{num}[4] && \text{by D-Plus} \end{aligned}$$

$$\begin{aligned} e_1(e_2) &\xmapsto{\text{na}}_{\mathrm{n}} e_1(u) && \text{by D-Print and D-App-2} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} u(u(\mathsf{num}[2])) && \text{by D-Print and D-App-2} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} u(\mathsf{print}(\text{`batman'}; \mathsf{plus}(\mathsf{num}[2]; \mathsf{num}[1]))) && \text{by D-App-2} \\ &\xmapsto{\text{batman}}_{\mathrm{n}} u(\mathsf{plus}(\mathsf{num}[2]; \mathsf{num}[1])) && \text{by D-Print and D-App-2} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} u(\mathsf{num}[3]) && \text{by D-Plus and D-App-2} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} \mathsf{print}(\text{`batman'}; \mathsf{plus}(\mathsf{num}[3]; \mathsf{num}[1])) && \text{by D-Beta} \\ &\xmapsto{\text{batman}}_{\mathrm{n}} \mathsf{plus}(\mathsf{num}[3]; \mathsf{num}[1]) && \text{by D-Print} \\ &\xmapsto{\varepsilon}_{\mathrm{n}} \mathsf{num}[4] && \text{by D-Plus} \end{aligned}$$

2. (∗) Do you need to prove progress and preservation for the CBV STLC, or can you say that they have already been established? Do not do any proofs in the answer to this question (unless you want to).

Solution: The dynamics of the language have changed, so there must be at least some small adjustments to the previous proofs.

Take preservation, which is proved by induction on $\longmapsto$. It must certainly be proved for the new rules D-Inl, D-Inr, and D-App-2. All these cases follow previous patterns, and are easy to show. It is debatable whether it needs to be shown for the rules that have changed, as these just limit previous reductions to make sure that substitutions happen only for values, and values are a subset of pre-terms—so the previous proofs essentially apply.

In the case of progress, things change a little more. Progress is the statement that any well-typed term is either a value, or a transition out of it can be taken by the dynamics. The typing rules have not changed, so the shape of the inductive proof is more or less the same. However, the available transitions have, and this might change some of the inductive cases, e.g. the one for terms of the form $e_1(e_2)$.