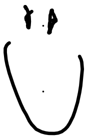
~mih Sheet~



Dynamics - Values

Our programs will compute values

$$\text{VAL-NUM}\ \frac{n \in \mathbb{N}}{\mathrm{num}[n]\ \mathsf{val}}$$

$$\text{VAL-STR}\ \frac{s \in \Sigma^*}{\mathrm{str}[s]\ \mathsf{val}}$$

If e val then either $\vdash e : \mathrm{Num}$ or
$\vdash e : \mathrm{Str}$

Values are closed expressions

Dynamics - Transitions

We are going to use <u>operational</u> semantics

Transition system:
- closed terms = states
- instruction transitions, which perform computation
- search transitions, which determine the eval order of the lang

Rules:

$$\text{D-PLUS}\quad \frac{n_1 + n_2 = n}{\text{plus}(\text{num}[n_1];\text{num}[n_2]) \mapsto \text{num}[n]}$$

$$\text{D-PLUS-1}\quad \frac{e_1 \mapsto e_1'}{\text{plus}(e_1;e_2) \mapsto \text{plus}(e_1';e_2)}$$

$$\text{D-PLUS-2}\quad \frac{e_1\ \text{val} \qquad e_2 \mapsto e_2'}{\text{plus}(e_1;e_2) \mapsto \text{plus}(e_1;e_2')}$$

$$\left(\text{D-PLUS-2'}\quad \frac{e_2 \mapsto e_2'}{\text{plus}(e_1;e_2) \mapsto \text{plus}(e_1;e_2')}\right)$$

$$\text{D-LEN-1}\quad \frac{e \mapsto e'}{\text{len}(e) \mapsto \text{len}(e')}$$

$$\text{D-LEN}\quad \frac{|s| = n}{\text{len}(\text{str}[s]) \mapsto \text{num}[n]}$$

$$\text{D-LET}\quad \frac{}{\text{let}(e_1;x.e_2) \mapsto e_2[e_1/x]}$$

plus(len(str['asdf']);num[1])
$\mapsto$ ①
plus(num[4];num[1])

$$\text{D-PLUS-1}\quad \dfrac{① \quad \text{D-LEN}\ \dfrac{|\text{'asdf'}| = 4}{\text{len}(\text{str}[\text{'asdf'}]) \mapsto \text{num}[4]}}{\text{plus}(\text{len}(\text{str}[\text{'asdf'}]);\text{num}[1]) \mapsto \text{plus}(\text{num}[4];\text{num}[1])}$$

Dynamics - Multi-step transitions

$\text{plus}(\text{len}(\text{str}[\text{'asdf'}]); \text{num}[1])$
$\overset{①}{\longmapsto}$
$\text{plus}(\text{num}[4]; \text{num}[1])$
$\longmapsto$
$\text{num}[5]$

Transition sequence

reflexive transitive closure of $\mapsto$

every elem is related to itself (no $\mapsto$ steps)

relations can be chained together (adding a $\mapsto$ step to our sequence)

$\mapsto$ single step

$\mapsto^*$ multi steps

D-MULTI-REFL $\dfrac{}{e \mapsto^* e}$

D-MULTI-STEP $\dfrac{e \mapsto e' \quad e' \mapsto^* e''}{e \mapsto^* e''}$

$\boxed{\ggg}$ $\ggg^*$

$\dfrac{}{e \ggg^* e}$ $\ggg$REFL

$\dfrac{e \ggg e' \quad e' \ggg^* e''}{e \ggg^* e''}$ $\ggg$STEP

Dynamics - Properties

(Finality) If e **val** then there is no e' with $e \mapsto e'$

Proof by inspection.

(Determinism) If $e \mapsto e_1$ and $e \mapsto e_2$ then $e_1 \equiv e_2$ (up to α-equiv)

(TLC: $\rightarrow_\beta$ confluence.)

Notation:

$$\underbrace{e \Downarrow v}_{\text{e evals to v}} \stackrel{\text{def}}{\equiv} \underbrace{e \mapsto^{*} v}_{\text{there is a transition sequence from e to v}} \wedge \underbrace{v \textbf{ val}}_{\text{v is a value}}$$

Due to the determinism of our dynamics v is unique

Type Saftey

(Type Saftey)

1. (Preservation) If $\vdash e : T$ and $e \mapsto e'$ then $\vdash e' : T$

2. (Progress) If $\vdash e : T$ then either e val or $e \mapsto e'$ for some e'.

Preservation