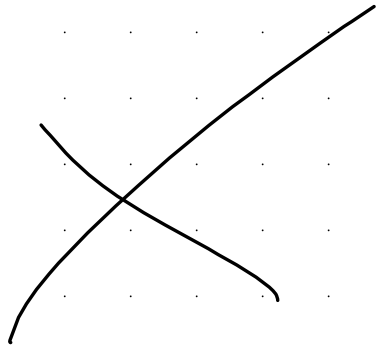
~min sheet~



Induction cont.

Induction for mutually and simultaneously defined rules

e.g. $$\frac{}{\text{zero even}}\ \text{EVEN-Z}$$

$$\frac{n \text{ odd}}{\text{succ } n \text{ even}}\ \text{EVEN} \qquad \frac{n \text{ even}}{\text{succ } n \text{ odd}}\ \text{ODD}$$

Induction principle

Let P be a property of even nums
Let Q be a property of odd nums

P/Q will hold for all even/odd nums if:

- P(zero)
- Whenever n even and P(n) we have Q(succ n)
- Whenever n odd and Q(n), we have P(succ n)

Recipe for proof by induction:

1. Decide who to induct on
   - You must induct on a premise
   - Pick the "most interesting" premise
2. State that you are doing a proof by induction and on which premise.
3. Complete your proof by cases
   - One case per rule
   - Assume your (other) premises
   - Write your IH

Example

premise

Claim: If succ(n) nat then n nat

conclusion

Proof:

Proof by induction on succ(n) nat

[Case: zero]

GOAL = If succ(zero) nat then zero nat

P = succ(zero) nat

GOAL' = zero nat

$$\frac{}{\text{zero nat}}\ \text{Z}$$

□zero

[Case: succ]

GOAL = if succ(succ n) nat then succ n nat

P = succ(succ n) nat

GOAL' = succ n nat

IH = if succ n nat then n nat

By P we have succ(succ n) nat, which must have had the following derivation:

$$\frac{\begin{array}{c}\vdots\\ \hline \text{succ } n \text{ nat}\end{array}}{\text{succ(succ } n\text{) nat}}\ \text{S}$$

The derivation of P contains a derivation of our goal

□succ

□

Example

Claim: If n even then either CLAIM-E
n = zero, or
n = succ x, where x odd

Proof:

Proof by simultaneous induction on n even

Complementary claim for odd numbers:

If n odd then n = succ(x), where x even

CLAIM-O

[Case: zero]

GOAL = If zero even then either
zero = zero, or
zero = succ(x), where x odd

This is immediate as n = zero.

□zero

[Case: Even]

GOAL = if succ n even then either
succ n = zero, or
succ n = succ x, where x odd

P = succ n even

GOAL' = succ n = succ x, where x odd

IH = CLAIM-O for n
= if n odd then n = succ x, where x even

$$\frac{\begin{array}{c}\vdots\\ \hline n \text{ odd}\end{array}}{P = \text{succ } n \text{ even}}\ \text{EVEN}$$

□even

[Case: ODD]

Exercise.