Welcome!

Claims + Evidence

PL Proofs
- logical
- computable / decidable

⇒ constructive / intuitionistic logic

Judgement (claim):

2 nat

Rules (evidence):

```
> data Nat = Z
>          | S Nat

> two :: Nat
> two = S (S Z)

> three = S two
>       = S (S (S Z))
```

$$\frac{}{\text{zero nat}}\ Z$$

$$\frac{n\ \text{nat}}{\text{succ } n\ \text{nat}}\ S$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{\text{zero nat}}\ Z}{\text{succ zero nat}}\ S}{\text{succ (succ zero) nat}}\ S}{\text{succ (two) nat}}\ S$$

```
> data Even = Zero
>           | Even Odd
> data Odd  = Odd Even

> one : Odd
> one = Odd zero

> two = Even one
```

$$\frac{}{\text{zero even}}\ \text{EVENZ}$$

$$\frac{n\ \text{odd}}{\text{succ } n\ \text{even}}\ \text{EVEN}$$

$$\frac{n\ \text{even}}{\text{succ } n\ \text{odd}}\ \text{ODD}$$

$$\cfrac{\cfrac{\cfrac{}{\text{zero even}}\ \text{EVENZ}}{\text{succ zero odd}}\ \text{ODD}}{\text{succ(succ zero) even}}\ \text{EVEN}$$

Derivable + Admissible Rules

```
>succsucc :: Nat -> Nat
>succsucc n = S(S n)
```

$$\dfrac{\vdots}{n\ nat}$$

$$\dfrac{n\ nat}{succ(succ\ n)\ nat}\,SS$$

A rule is derivable if we can use a derivation of its premise as a building block in deriving its conclusion

$$\dfrac{\dfrac{\boxed{\dfrac{\vdots}{n\ nat}}}{succ\ n\ nat}\,S}{succ\ (succ\ n)\ nat}\,S$$

An admissible rule is rule where if we have a derivation of its premise, we know that we can construct a derivation of the conclusion

$$\dfrac{succ\ n\ nat}{n\ nat}\,NS$$

```
>match :: Nat -> Nat
>match (S n) = n
```

n = succ zero

$$\dfrac{\dfrac{\dfrac{}{zero\ nat}\,Z}{succ\ zero\ nat}\,S}{succ(succ\ zero)\ nat}\,S \qquad \dfrac{\dfrac{}{zero\ nat}\,Z}{succ\ zero\ nat}\,S$$

Induction

$$Z\ \frac{}{\text{zero nat}} \qquad S\ \frac{n \text{ nat}}{\text{succ } n \text{ nat}}$$

P is a property that holds for all nats if

- If it holds for zero (P(zero))
- Whenever it holds for n (P(n)), it also holds for succ n (P(succ n))

$$\frac{}{\text{zero even}}\ \text{EVEN Z}$$

$$\frac{n \text{ odd}}{\text{succ } n \text{ even}}\ \text{EVEN}$$

$$\frac{n \text{ even}}{\text{succ } n \text{ odd}}\ \text{ODD}$$